Econ 511b | Spring 1999 | G.Hall and C.Sims

Final Examination

You have 3 hours (180 minutes) to complete the exam. Answer all the questions. The number of minutes suggested below for each question add up to 150. Points will be given in grading in proportion to the suggested minutes.

1. (15 minutes) Explain the statement "first-order deviations in decision rules have second-order consequences for utility." Does second-order necessarily mean small? Why or why not?
2. (15 minutes) Explain what a "no-Ponzi" condition is, what a transversality condition is, and what the difference between them is.
3. (30 minutes) Consider an economy with a continuum of identical households. Let (U, y, x, ξ) be, respectively, the unemployment rate, the inflation rate, the average across households of the expected inflation rate, and the inflation rate expected by a particular household. The government chooses $y \in Y$ where $Y = [0, y^{\#}]$. Each household chooses ξ, taking U, Y, and x as given, and the average over the households' choice of ξ is x. The unemployment rate is pinned down by a 'natural-rate Phillips curve'

$$U = U^* - \theta(y - x), \quad \theta > 0.$$

Each household has a payoff function:

$$-.5[(y - \xi)^2 + y^2].$$

The government has the following loss function:

$$-.5(U^2 + \gamma y^2).$$

(a) For this one-period economy define the following objects:
  (i) a *competitive rational expectations equilibrium*,
  (ii) the *Ramsey problem* (i.e. the problem when commitment is possible for the government),
  (iii) the *Ramsey outcome*,
  (iv) a *best response function* for the government, and
  (v) a *Nash equilibrium*.

(b) Compute the best response function for the government, the Nash equilibrium, and the Ramsey outcome for this one-period economy. How do the Nash equilibrium and Ramsey outcome vary with the government's distaste for inflation, γ?

4. (30 minutes) Consider a model in which representative agents maximize

$$E\left[\sum_{t=0}^{\infty}\beta^t\left(\log C_t+\log(1-L_t)\right)\right] \tag{1}$$

subject to

$$C_t+\frac{B_t}{P_t}=R_{t-1}\frac{B_{t-1}}{P_t}-\tau_t+w_tL_t+\pi_t \tag{2}$$

$$B_t\geq 0\,. \tag{3}$$

The agents choose C_t, L_t and B_t at t, taking the process generating P_t, τ_t, w_t and R_t as beyond their control. They know as they choose variables dated t the values of all the other variables in the model dated t and earlier.

The government has a constraint

$$\frac{B_t}{P_t}=R_{t-1}\frac{B_{t-1}}{P_t}-\tau_t \tag{4}$$

and can be thought of as choosing two of the variables that enter its constraint. We take government policy to set $R_t\equiv\bar{R}$ and to set $\tau_t=\bar{\tau}+\varepsilon_t$, where ε is an i.i.d. random disturbance.

Firms maximize

$$E\left[\sum_{t=0}^{\infty}\beta^t\Phi_t\pi_t\right] \tag{5}$$

subject to

$$\pi_t=\frac{P_t}{\bar{P}_t}L_t^{\alpha}-w_tL_t-\xi\left(\frac{P_t-P_{t-1}}{\bar{P}_{t-1}}\right)^2 \tag{6}$$

$$\frac{L_t^{\alpha}}{\bar{L}_t^{\alpha}}=\left(\frac{P_t}{\bar{P}_t}\right)^{-\theta}\,. \tag{7}$$

The firms take the paths of the stochastic discount factor Φ and of the aggregate price index $\bar{P}$ as beyond their control. The firms choose π, L, and P. As usual they know exogenous variables dated t and earlier when they choose variables dated t. [Each component below gets 5 minutes' weight, except that (4d), where you find FOC's, gets 10 minutes' weight.]

(a) This is a monopolistic competition/menu cost model. Which term in which equation makes it a monopolistic competition model? Which term in which equation makes it a menu cost model?

(b) This model will generally not show price neutrality. That is, the allocation of real quantities will vary across equilibria in which there are the same technological conditions and agent preferences, but prices follow different paths. By setting a certain parameter in the model to zero, though, we can make it a model that shows strict neutrality. Which parameter is that?

(c) This is a model of an economy with no money. Explain in words what determines the price level in a model like this one without money.

(d) Find the first order conditions for the consumer and firm and any other equations or conditions needed to determine competitive equilibrium in this economy.

(e) We will assume that the Φ_t sequence of stochastic discount factors satisfies

$$\frac{\Phi_{t+1}}{\Phi_t} = \frac{C_t}{C_{t+1}} \tag{8}$$

for all t. This can be justified as what would emerge from the presence of complete capital markets. Explain briefly how this condition is related to the existence of complete capital markets.

5. (30 minutes) [Though this question uses the model of question 4, it can be answered independently.] In the model you discussed in question 4, it is possible to linearize FOC's, constraints, and market clearing condition and reduce the system to the following form, in which we set $y_t = [B_t, p_t, p_{t-1}, \ell_t]'$, $z_t = \varepsilon_t$, and η_t is a 2×1 vector of endogenously determined disturbances satisfying $E_t \eta_{t+1} = 0$:

$$\Gamma_0 y_t = \Gamma_1 y_{t-1} + \Psi z_t + \Pi \eta_t \,. \tag{9}$$

The lower case letters p and ℓ correspond to logs of the upper case letters in problem (4).

(a) If we set $\beta = 1/1.05$, $\bar{R} = 1.05$, $\alpha = .5$, $\theta = 2$, $\xi = .1$, and $\bar{\tau} = 1$, we get a matrix $\Gamma_0^{-1}\Gamma_1$ whose eigenvalues are 1.0500, 1.0000, -0.4862, -2.1595. If, on the other hand, we use the same values for all parameters except ξ, changing ξ to .2, the eigenvalues are instead 1.0500, 1.0000, -0.1489+1.0138i, -0.1489-1.0138i. This is enough information for us to determine that (unless we are in a knife-edge special case, which we are not) one of these parameter settings implies the existence of a unique nonexplosive solution, while the other does not. Which is which? What fails in the "bad" parameter setting, existence? uniqueness? both? [We are excluding as "explosive" any equilibrium in which variables grow at a positive exponential rate.]

(b) We can drop the government budget contraint from the model and delete the B variable from the system. This leaves us with three equations in the three variables p_t, p_{t-1}, and ℓ_t. It turns out that in this reduced system, which still contains the same two-dimensional η vector, the list of eigenvalues for $\Gamma_0^{-1}\Gamma_1$ is the same as before, except that the root of 1.05 disappears from the list in both cases. Explain why this appears to reverse your previous conclusion about which parameter setting implies existence of a unique solution.

(c) Is one of these versions of the model (the 4-variable one including the government budget constraint or the 3-variable one excluding it) giving us the right conclusion and the other the wrong one? What is the economic interpretation of these apparently contradictory results?

6. (30 minutes) Consider an economy with a continuum of *ex ante* identical households (with mass 1) each of whom evaluates consumption streams according to

$$E \sum_{t=0}^{\infty} \beta^t u(c_t).$$

Every period each household draws an employment opportunity, s_t, which can take on one of two values: employed or unemployed. Assume s_t evolves according to a two state Markov chain with a transition matrix $\mathcal{P}$ where:

$$\mathcal{P}(s, s') = \text{Prob}\left(s_{t_1} = s' \middle| \, s_t = s \right) \text{ for } s, s' \in \{\text{employed,unemployed}\} \, .$$

Each period the household receives $y(s_t)$ in labor income. This household specific randomness is distributed identically and independently across households, though for a given household there is dependence across time. In other words, there is no aggregate uncertainty in this economy.

Assume agents can borrow and lend risk free among themselves at a gross interest rate, R_t, so that the individual's one-period budget constraint is

$$W_t = R_{t-1} W_{t-1} + y(s_t) - c_t \, .$$

For an individual agent, borrowing cannot exceed F, i.e. $W_t \geq -F$. Agents know their current employment state s_t, but not s_r for $r > t$, when they choose c_t.

Because each household will have its own idiosyncratic history of employment states, there will generally be a distribution of net worth across agents, which will differ between unemployed and employed agents.

(a) Write down the household's Bellman equation.

(b) Define a stationary competitive equilibrium for this economy.

(c) Describe an algorithm to compute this stationary equilibrium. (Do not try to implement this algorithm. It could take more computing power than you have access to at the moment.)